

CHOICE, CONTINGENCY DISCRIMINATION, AND FORAGING THEORY

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Four pigeons were trained on eight or nine pairs of independent concurrent variable-interval schedules. The range of reinforcement ratios included extreme ratios (up to 532 to 1). Large samples of stable performance were gathered. Contrary to the findings of Davison and Jones (1995), the generalized matching law described choice more accurately than a contingency-discriminability model. Taking small samples (5 to 10 sessions) and applying a more liberal stability criterion used by Davison and Jones only increased the unsystematic variance in the data and in estimates of generalized-matching-law sensitivity. Because changing to dependent scheduling and inserting a changeover delay had no systematic effect, the deviations from generalized matching reported by Davison and Jones probably arose from imperfectly discriminated stimuli. Analysis of visits revealed that visits to the nonpreferred alternative were brief and approximately constant. When choice between the preferred (rich) and nonpreferred (lean) alternatives, regardless of position, was analyzed according to the generalized matching law, sensitivities approximated 1.0, with bias in favor of the lean alternative. This bias, which arose from an excessive frequency of visits to the lean alternative, explains undermatching as the result of fitting one line to a choice relation that consists of two displaced lines, both with a slope of 1.0. The pattern of deviation from the generalized matching line confirmed this account. The findings suggest an alternative analysis of choice that focuses on probability of visiting the lean alternative as the dependent variable. This probability was directly proportional to ratio of reinforcement. Matching, undermatching, and overmatching may all be explained by a view of concurrent performance based on foraging theory, in which responding occurs primarily at the rich alternative and is occasionally interrupted by brief visits to the lean alternative.

Key words: choice, generalized matching law, contingency-discriminability model, foraging theory, visit duration, key peck, pigeons

Choice has been studied in the experimental analysis of behavior with concurrent schedules of reinforcement. The matching law (Herrnstein, 1961) was first proposed to describe performance on such schedules. The matching relation equates the proportion of responses at an alternative with the proportion of reinforcement obtained from that alternative:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2}, \quad (1)$$

where B_1 and B_2 are the rates of responding and r_1 and r_2 are the rates of reinforcement obtained from Alternatives 1 and 2. This equation was later put into the algebraically

equivalent ratio form (Baum & Rachlin, 1969):

$$\frac{B_1}{B_2} = \frac{r_1}{r_2}. \quad (2)$$

As an attempt to explain systematic deviations from the "strict" matching of Equation 2, Baum (1974b) offered the generalized matching law:

$$\frac{B_1}{B_2} = b \left(\frac{r_1}{r_2} \right)^s. \quad (3)$$

The parameter b describes bias, or preference due to differences in variables other than r_1 and r_2 . When b equals 1.0 there is no bias. If b is greater or less than 1.0, there is a bias toward Alternative 1 or 2. The parameter s has been interpreted as sensitivity to the reinforcement ratio (Lobb & Davison, 1975). When s falls short of 1.0, a result called undermatching, choice tends to be more indifferent than predicted by strict matching, whereas when s exceeds 1.0, the result called overmatching, choice is more extreme than predicted by strict matching (Baum, 1979).

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Performance under concurrent pairs of variable-interval (VI) schedules has commonly been described by the generalized matching law in its logarithmic form (for reviews, see Baum, 1979; Davison & McCarthy, 1988; Wearden & Burgess, 1982), which is linear:

$$\log\left(\frac{B_1}{B_2}\right) = s \cdot \log\left(\frac{r_1}{r_2}\right) + \log b, \quad (4)$$

where $\log b$ is the intercept and sensitivity s is the slope. Within subjects, sensitivity is assumed to be constant across ratios of reinforcement.

For concurrent VI schedules, the value of s has often been found to deviate from 1.0 (Baum, 1979; Wearden & Burgess, 1982). In proposing Equations 3 and 4, Baum (1974b) speculated that several factors might affect sensitivity, such as penalties for switching between alternatives and failure of discrimination between alternatives. Although undermatching occurs frequently, its origins remain a puzzle. Penalties on switching, however, are well known to increase sensitivity, even to overmatching (Baum, 1975, 1982; Boelens & Kop, 1983).

Several mathematical models have been proposed to explain matching. For example, Shimp (1966) proposed momentary maximizing, Myerson and Miezin (1980) proposed a kinetic model, Herrnstein and Vaughan (1980) proposed melioration, Hinson and Staddon (1983) proposed hill climbing, and Rachlin (1978) proposed global optimization. Although all of the theories that have been proposed explain matching, few explain the frequently observed deviations—undermatching and overmatching.

One attempt to explain undermatching was put forward by Davison and Jenkins (1985). Elaborating on the possibility of failure of discrimination, they proposed that undermatching might be explained by misallocation of reinforcers between alternatives (but see Wearden, 1983, for an alternative interpretation). They argued that failure of discrimination between the stimuli associated with the different alternatives would result in some of each alternative's reinforcers being misallocated to the other. An experiment by Miller, Saunders, and Bourland (1980) lent support to this idea by showing that in a single-key procedure (Findley, 1958) response

distribution depended on the disparity between the stimuli associated with the different component schedules and that undermatching increased as stimulus disparity decreased.

Hoping to explain undermatching and replace the generalized matching law, Davison and Jenkins (1985) proposed the contingency-discriminability model, which may be written

$$\frac{B_1}{B_2} = b \left(\frac{r_1 - pr_1 + pr_2}{r_2 - pr_2 + pr_1} \right). \quad (5)$$

Equation 5 includes two free parameters, b and p . The parameter b describes bias and is identical to parameter b of Equation 3. The parameter p replaces s in Equation 3. Whereas s can be thought of as sensitivity to the ratio of reinforcement from the two sources, p is interpreted as proportional confusion between the two sources of reinforcement (Davison & Jones, 1995). It incorporates the idea that whenever the stimuli associated with schedules of reinforcement are imperfectly discriminable, some proportion of the reinforcers obtained from each schedule are associated with the alternative schedule and its discriminative stimulus. With two concurrent schedules, p can range from 0 to 0.5. When b equals 1.0 and p is zero (indicating no confusion between alternatives) the contingency-discriminability model simplifies to strict matching (Equation 2). When p equals 0.5, the alternatives are perfectly confused, and the equation predicts no change in choice as reinforcement changes. Because p is a proportion, values of p between 0 and 0.5 dictate that, as the ratio of reinforcement becomes more extreme, more and more reinforcers obtained from the rich schedule are associated with the lean schedule. Hence, the curve defined by Equation 5 appears S-shaped in logarithmic coordinates, but for moderate values of p , the curve is approximately linear over the range of reinforcement ratios usually studied (between 10 and 0.1). Thus, the predictions of the contingency-discriminability model and the generalized matching law differ significantly only when the reinforcement ratio becomes more extreme than usual (e.g., ratios greater than 10:1 or less than 1:10).

Davison and Jones (1995) set out to test the contingency-discriminability model by studying choice with reinforcement ratios that var-

ied over a wider range than usual: up to about 100:1 and down to about 1:100. They found that the generalized matching law (Equation 4) accounted for about 98% of the variance in choice and that the contingency-discriminability model (Equation 5) accounted for about 99% of the variance. They argued that the difference was significant after fitting the generalized matching law both with and without the most extreme ratios of reinforcement. When the extremes were eliminated the slope (s in Equation 4) increased, indicating that choice fell short at extreme reinforcement ratios, as the contingency-discriminability model predicted.

Their results, however, may have arisen from specific aspects of the procedures that they used. In particular, Davison and Jones (1995) used dependent scheduling (Stubbs & Pliskoff, 1969), in which the set-up of a reinforcer for one alternative causes the schedules for both alternatives to stop until the set-up reinforcer is obtained, and a 3-s changeover delay (COD), which blocked reinforcement after a switch of schedules until at least 3 s had elapsed. Both of these would tend to add responses to the lean alternative, reducing relative responding at the rich alternative, particularly at the extremes.

More important, Davison and Jones (1995) used unusually confusable stimuli. They arranged a single-key procedure (Findley, 1958), in which the two schedules were associated with two different brightness levels on the main key. More typical procedures associate the schedules with highly distinctive stimuli, such as color or location. The present experiment made a more conservative test of the generalized matching law by associating the two schedules with two separate keys, a more typical method of presenting concurrent schedules.

Finally, Davison and Jones (1995) used a stability criterion that may have been too liberal and samples of stable performance that may have been too small. When reinforcement ratios reach 100:1 or more, several sessions can pass without a reinforcer on the lean alternative. Their stability criterion allowed conditions to change when only 10 or fewer reinforcers had been obtained at the lean alternative in total. They then used the last five sessions as their sample, with the result that their samples from the extreme con-

ditions included fewer than five reinforcers on the lean alternative. Small samples might mean just that estimates of response and reinforcer ratios would be more variable at the extremes; if so, the deviations observed by Davison and Jones might simply have been due to chance.

Davison and Jones (1995) claimed that, whereas the generalized matching law offers only a description, the contingency-discriminability model offers an explanation, in the sense that it derives undermatching from assumptions about basic principles (e.g., confusion). Equation 5 has the limitation, however, that it only predicts undermatching. It cannot explain overmatching because, for that, p would have to take on negative values, an outcome with no meaning.

Houston and McNamara (1981; Houston, McNamara, & Sumida, 1987) proposed a model that has the potential to predict the full range of variation in sensitivity, from undermatching to overmatching. It derives from foraging theory, in which foraging and choice are seen as composed of successive visits to alternative sites where resources may be obtained. Accordingly, concurrent performance would consist of visits, now to one schedule, now to the other. Houston and McNamara derived an optimal performance that consists of responding primarily at the rich alternative interrupted by brief visits to the lean alternative. At low rates of switching—the optimum—this predicts overmatching. Higher rates of switching, though suboptimal, would lead to reductions in sensitivity, first to matching and then to undermatching.

Casting concurrent performance in terms of rich and lean alternatives rather than in terms of locations or stimuli, foraging theory points to a different sort of analysis of choice. Although bias in Equations 4 and 5 is usually taken to indicate favoritism toward one position or stimulus, independent of which is rich or lean, it is possible to think of another sort of bias: favoritism toward the rich or lean alternative, independent of position or stimulus. A bias in favor of the lean alternative would mean too frequent visiting of the lean alternative and would appear as undermatching in a fit of the generalized matching law. A bias in favor of the rich alternative would appear as overmatching.

Depending on availability of resources, cost

of switching from one resource site to another, and the way visits are aggregated, a foraging model may predict the whole range of observed outcomes when choice, measured as relative aggregated visits, is considered as a function of relative resources. Departing from generalized matching, however, it suggests other analyses that focus on visits to the lean alternative. Some research favors such a possibility (Baum, 1982; Baum & Aparicio, 1999; Buckner, Green, & Myerson, 1993). Particularly when choice and resource ratios vary over a wide range, from near indifference to extremes, it seems possible to put a model like this to the test.

The present experiment used a conventional set of procedures to study choice over an extremely wide range. It had two aims: (a) to compare the predictions of the generalized matching law and the contingency-discriminability model, and (b) to test the validity and implications of the Houston-McNamara view of concurrent performance as composed of visits to sources of reinforcement.

METHOD

Subjects

Four White Carneau pigeons, numbered 26, 27, 299, 973, participated. Pigeon 299 was maintained at 85% of free-feeding weight ± 15 g and was experimentally naive at the beginning of the experiment. The other 3 birds were maintained at 80% of free-feeding weights (± 15 g), and all had previous experience with a variety of concurrent schedules. After each daily session the birds were weighed and, if necessary, were fed an amount of pigeon chow sufficient to maintain their designated body weights. Water and grit were constantly available in the home cages.

Apparatus

Four typical three-key operant conditioning chambers (35 cm deep by 35 cm wide by 35 cm high) were used. The circular keys (2.5 cm diameter) were aligned horizontally 26 cm above the floor and were transilluminated with white light. Each chamber was equipped with a houselight 7 cm above the center key, and an aperture (6 cm by 5 cm) that allowed access to a grain magazine was located 13 cm below the center key. The magazine was illuminated when operated, at which time wheat

was available. A force of approximately 0.10 N was required to operate a key. Operation resulted in an audible feedback click. Each chamber was enclosed in a sound-attenuating box and was fitted with a ventilating fan, which helped to mask extraneous noises. Event scheduling and data recording were controlled by a microcomputer located in the next room.

Procedure

Pigeon 299 was gradually deprived of food before being trained to eat from the food magazine. An autoshaping procedure was then used to train the bird to peck keys transilluminated by the white light. When it was pecking reliably, it was trained on a Fixed-Ratio 2 schedule for two 30-min sessions. Then the bird was trained with a VI 30-s schedule on the center key before concurrent VI 30-s schedules on the left and right keys were introduced. All birds were exposed to concurrent VI 30 VI 30 for several days before Condition 1. All VI schedules were random-interval schedules with a 0.5-s time base and differed only in the probability of setting up reinforcement.

During the session both the left and right keys were transilluminated with white lights of approximately equal intensity. Pecks to the keys were reinforced on independent concurrent VI schedules. The overall programmed reinforcement rate was held constant at 2 per minute. During reinforcement the keylights were darkened and the food magazine was raised and illuminated. Reinforcement duration was slowly adjusted separately for each bird during the first and second conditions from an initial duration of 2.00 s so that postsession feeding could be kept at a minimum. Pigeons 26, 27, 299, and 973 were allowed to eat for 2.39, 2.00, 2.50, and 2.07 s, respectively. Sessions commenced in blackout and lasted until 80 reinforcers had been obtained (usually about 45 min). If a reinforcer had become available on one of the schedules but was uncollected at the end of the session, the next day's session began with a reinforcer available on that schedule.

No changeover delay was used. This was done to avoid forcing extra responses to the lean alternative and to avoid ambiguity about the status of responses made during the COD. This ambiguity arises because the COD

Table 1

The order of conditions, scheduled reinforcement ratios, and sessions per condition for each subject.

Order	Pigeon 26		Pigeon 27		Pigeon 299		Pigeon 973	
	Scheduled ratio	Sessions	Scheduled ratio	Sessions	Scheduled ratio	Sessions	Scheduled ratio	Sessions
1	4:1	27	1:4	27	1:4	27	1:4	27
2	1:9	29	9:1	36	1:9	24	9:1	24
3	64:1	37	1:64	34	64:1	46	1:64	42
4	1:32	44	32:1	40	1:32	29	32:1	36
5	128:1	44	1:128	50	128:1	55	1:128	65
6	1:256	90	256:1	147	1:256	137	256:1	144
7	4:1	47	1:4	49	4:1	47	1:4	52
8	1:8	27	8:1	44	1:8	54	8:1	54
9	64:1	50	1:128 d	67	128:1 d	54	1:128 d	79
10	1:256 d	77	256:1 d	91	1:256 d	111	256:1 d	62
11	128:1 d	66	1:128 d,c	75	128:1 d,c	66	1:128 d,c	75
12	1:256 d,c	53	256:1 d,c	69	1:256 d,c	78	256:1 d,c	69
13	128:1 d,c	58						
14	1:256 d,c	78						

Note. A d indicates dependent scheduling; a c indicates the use of a 3-s COD.

is discriminated: Responding during the COD occurs at a high rate that differs little between alternatives (Baum, 1974a). As a result, responding during the COD exhibits extreme undermatching. Because post-COD responding tends toward overmatching, the common practice of combining the two as if they were equivalent produces a result somewhere in between, often approximating matching (Baum, 1974a). It is unclear, however, that they are equivalent. It may be argued that the COD resembles travel between alternatives and should be omitted from calculations of choice (Baum, 1982). These considerations, combined with findings that matching may be obtained without a COD (e.g., Heyman, 1979; Shull & Pliskoff, 1967), favored omission of the COD.

Table 1 shows the order of conditions, scheduled ratio of reinforcement, and total number of sessions in each condition for each bird. Each condition remained in effect for a pigeon until the logarithm of the behavior ratio appeared stable across sessions by visual inspection and a minimum of 10 reinforcers had been obtained from the lean schedule over the period of stable performance. In practice, this required a minimum of 24 and a maximum of 147 sessions. With one exception (Pigeon 299, Condition 2), when conditions were changed, the positions of the rich and lean schedules were always

reversed. Except for Pigeon 299, the first condition was replicated for every bird.

Following the initial set of eight or nine conditions, some additional extreme conditions were conducted for each bird, two or three with dependent scheduling and then two or three with dependent scheduling and a 3-s COD.

RESULTS

To compare the contingency-discriminability model with the generalized matching law, three separate analyses of choice were performed for each bird. The first used the data from the longest period of stability, as determined by visual inspection. The numbers of responses, reinforcers, and changeovers were summed across sessions. These numbers, along with the obtained ratios of reinforcement for each condition and pigeon, appear in the Appendix. The second analysis used sums of responses and reinforcers from just the last five sessions, as did the analysis done by Davison and Jones (1995). In a few conditions, when the last five sessions included no reinforcer on the lean side, up to 3 more days were included to provide a calculable reinforcement ratio. Because all the conditions in the present experiment were continued beyond the number of sessions at which they would have terminated according to the sta-

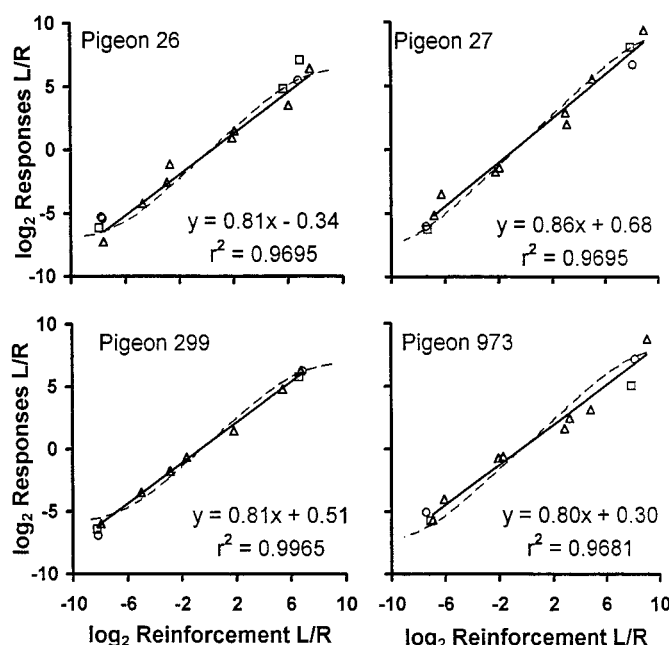


Fig. 1. Choice relations from data summed over the longest period of stability. \log_2 response ratios for each subject are plotted as a function of \log_2 obtained reinforcer ratios. The conditions with independent scheduling and no COD (Conditions 1 to 9 for Pigeon 26 and Conditions 1 to 8 for the other 3) are indicated by triangles. The conditions with dependent scheduling and no COD are indicated by squares. Circles indicate the conditions with dependent scheduling and a COD. The solid straight lines are the best fits of the triangles to Equation 4 by least squares regression. The equation and goodness of fit (r^2) are shown in each graph. The dashed curves show the best fits to Equation 5. The parameters of the fits are shown in Table 2.

bility criterion used by Davison and Jones, the third analysis attempted to assess the possible effects, had that stability criterion been applied to this experiment. Their criterion was used to pick the five sessions that would have been the last five sessions had that stability criterion been used to terminate conditions in this experiment. In a few conditions, up to five earlier sessions' data were included, as in the second analysis, to provide a calculable reinforcement ratio.

Figure 1 shows the logarithms (base 2) of the response ratios plotted as a function of the logarithms (base 2) of the obtained reinforcement ratios for each subject. Base 2 logarithms were used because, with them, a log unit represents a doubling or halving of ratio, and this allows distinctions among ratios to be seen more easily and in a more intuitive way than with base 10 logarithms. The only difference between base 2 and base 10 logarithms is multiplication by a constant; that is, they differ only in the size of the unit.

Figure 1 shows the best fits of Equations 4 and 5 to the points for the conditions with independent scheduling and no COD. Least squares regression was used to find the best fitting lines for the generalized matching law. Solver in EXCEL® was used to find the best fitting parameters, also by the method of least squares, of the contingency-discriminability model. The points for conditions with dependent scheduling, with or without a COD, deviate in no systematic way from the other points. Including them in the analysis produced no substantial changes in results.

Table 2 shows all of the parameters and proportion of variance accounted for (r^2) by both the generalized matching law and the contingency-discriminability model for each subject. For the large sample, all 4 birds showed undermatching, with sensitivity ranging from 0.80 to 0.86. For the contingency-discriminability model, the estimates of p were all low, ranging from 0.0024 to 0.0111. The biases greater than 1.0 indicate some

Table 2

Best fitting estimates of sensitivity (s) and bias (b) from the generalized matching law and proportion of confusion (p) and bias (b) from the contingency-discriminability model with maximum variance accounted for by each model for each of four types of analysis.

Pigeon	Generalized matching law			Contingency discriminability		
	s	b	r^2	p	b	r^2
26						
Large sample	0.81	0.79	.9695	.0086	0.82	.9431
Last five sessions	0.79	0.91	.9857	.0106	1.01	.9624
Davison-Jones	0.67	1.30	.9676	.0272	1.31	.9309
27						
Large sample	0.86	1.60	.9695	.0024	1.64	.9511
Last five sessions	1.00	1.86	.9441	-.0030	1.76	.9530
Davison-Jones	0.83	2.30	.9465	.0096	2.29	.9252
299						
Large sample	0.81	1.42	.9965	.0111	1.46	.9816
Last five sessions	0.70	1.42	.9688	.0237	1.41	.9727
Davison-Jones	0.88	1.54	.9940	.0061	1.64	.9858
973						
Large sample	0.80	1.23	.9681	.0041	1.26	.9209
Last five sessions	0.83	1.32	.9562	.0022	1.30	.9236
Davison-Jones	0.71	1.02	.9337	.0140	0.90	.9167

tendency to favor the left key. For all 4 pigeons, goodness of fit (r^2) was higher for the generalized matching law than for the contingency-discriminability model.

Figure 2 shows the results when the data were summed over only the last five sessions of each condition for each bird. This analysis produced sensitivity estimates that varied over a wider range: 0.70 to 1.00. Relative to the sensitivity values found for the large samples, sensitivity for Pigeons 26 and 973 remained about the same, for Pigeon 27 it increased, and for Pigeon 299 it decreased (see Table 2). Again the estimates of p in the contingency-discriminability model were small, and the one for Pigeon 27 was actually negative, an uninterpretable result. The results for goodness of fit were more variable than for the large samples: For 2 birds, r^2 was higher for the generalized matching law, and for 2 others, r^2 was higher for the contingency-discriminability model.

Figure 3 shows the results of applying the stability criterion employed by Davison and Jones (1995). After 11 sessions the relative response rates from sets of three sessions were assessed for monotonic trend. When a monotonic increase or decrease had been absent on at least five (not necessarily consecutive)

occasions, the data were summed over the last five sessions at the point at which Davison and Jones would have changed conditions. The results from Davison and Jones's stability criterion produced estimates of sensitivity that also were more variable, ranging from 0.67 to 0.88, than those found for the large samples (see Table 2). In addition, the sensitivity values for Pigeons 26, 27, and 973 decreased relative to the other two analyses. For those same 3 birds, the estimates of p were higher than for the other two analyses. For all 4 birds, goodness of fit (r^2) was higher for the generalized matching law.

For all three analyses, the fitted lines had slopes and bias values within the range of those typically found in concurrent VI VI performance (e.g., Baum, 1979; Taylor & Davison, 1983; Wearden & Burgess, 1982). Although both Equations 4 and 5 fitted the data well, Table 2 shows that in 10 of the 12 analyses the generalized matching law accounted for more variance than did the contingency-discriminability model.

The residuals, found by subtracting the obtained log response ratios from the predicted log response ratios of each model, were analyzed as a function of obtained log reinforcer ratios for each bird. If the choice ratios ap-

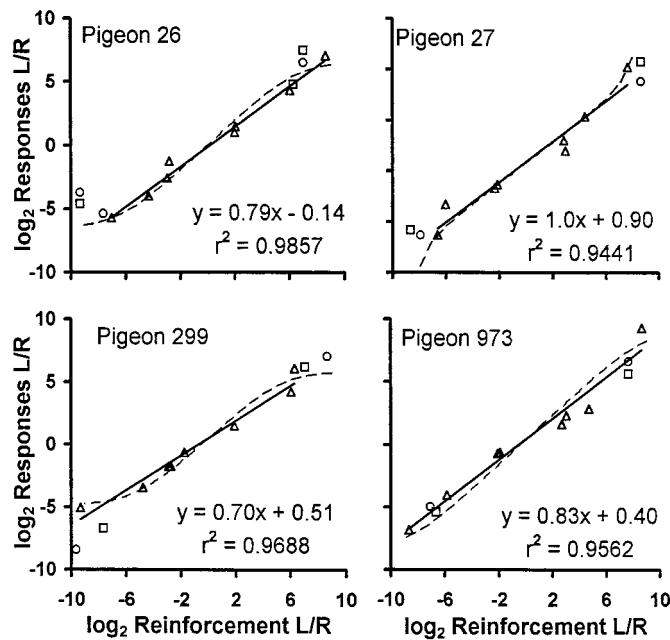


Fig. 2. Choice relations from data summed over the last five sessions of each condition. \log_2 response ratios for each subject are plotted as a function of \log_2 obtained reinforcer ratios. The solid straight line in each graph is the best fit to Equation 4 by least squares regression. The equation and goodness of fit (r^2) are shown in each graph. The dashed curves show the best fits to Equation 5. The parameters of the fits are shown in Table 2.

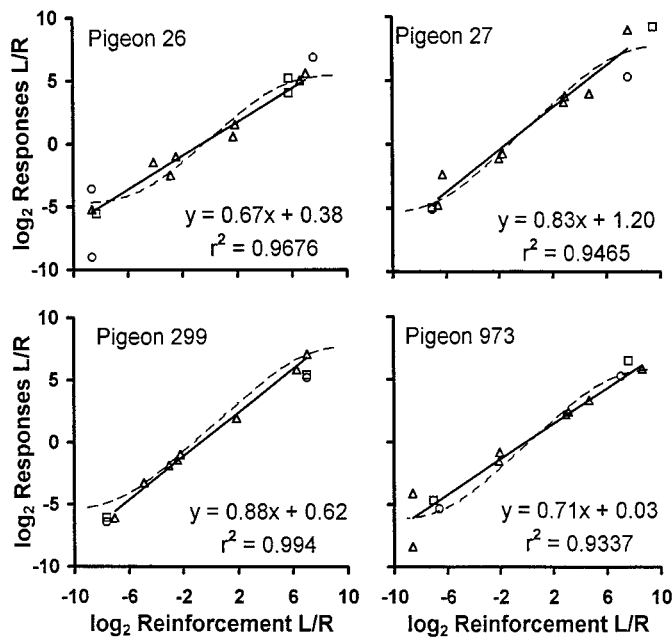


Fig. 3. Choice relations from data summed over five sessions based on Davison and Jones's (1995) stability criterion. \log_2 response ratios for each subject are plotted as a function of \log_2 obtained reinforcer ratios. The solid straight line in each graph is the best fit to Equation 4 by least squares regression. The equation and goodness of fit (r^2) are shown in each graph. The dashed curves show the best fits to Equation 5. The parameters of the fits are shown in Table 2.

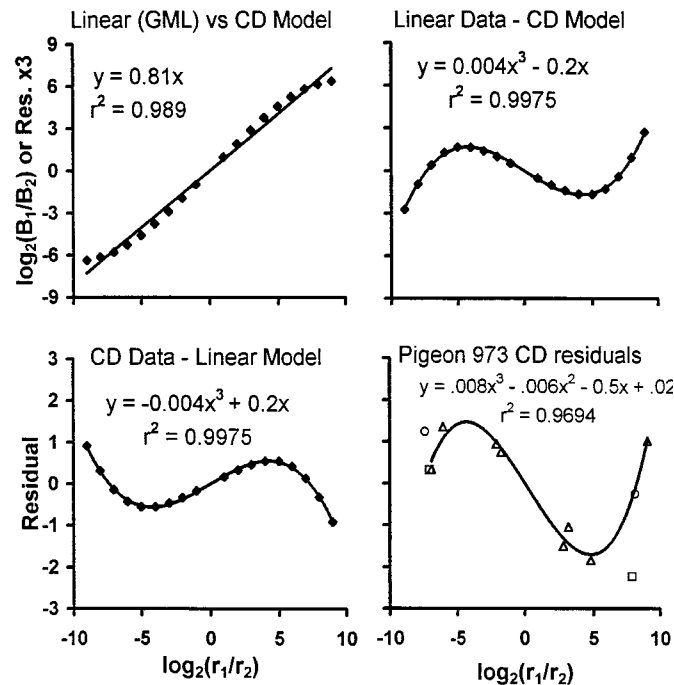


Fig. 4. Analysis of residuals. Upper left: generalized matching linear model fitted to choice ratios derived from the contingency-discriminability model with $p = .01$. Lower left: residuals from the upper left. The curve, fitted by least squares, is a third-order polynomial. Upper right: residuals of points on the line from the contingency-discriminability curve. Lower right: residuals of Pigeon 973's large samples (Figure 1) from the contingency-discriminability model. The third-order polynomial was fitted only to the triangles (independent VI schedules with no COD). Other symbols as in Figure 1.

proximated a straight line, the contingency-discriminability model would deviate from them systematically, producing a predictable pattern of residuals. A different pattern would result if the linear generalized matching relation were fitted to choice ratios that approximated the S shape required by the contingency-discriminability model. Figure 4 illustrates these patterns. The upper left panel shows points derived from the contingency-discriminability model with no bias and p equal to 0.01. The line was fitted by least squares to the points. Its slope (sensitivity) equals 0.81. Although the fit might be considered good ($r^2 = .99$), the points deviate systematically from the line. The lower left panel shows the residuals. At the extreme left, they are positive, they become negative at less extreme reinforcement ratios favoring Alternative 2, they become positive again for non-extreme reinforcement ratios favoring Alternative 1, and then they become negative again for extreme reinforcement ratios favoring Alter-

native 1. The curve shows a third-order polynomial that captured the pattern of the residuals almost perfectly ($r^2 = 1.0$). Only cubic and linear components were required; the quadratic component and zero-order constant both equaled zero. The upper right panel of Figure 4 shows the result of the opposite comparison—assuming the choice ratios to conform to a line while fitting the contingency-discriminability model to them. The result is exactly the opposite pattern, reflected in coefficients equal to the ones at the lower left but opposite in sign. These coefficients may be used to assess the patterns of the actually observed residuals.

If the situation is like that depicted in the two left panels of Figure 4, then we expect that the cubic coefficient will be negative and the linear coefficient will be positive. If the situation is the opposite—that the contingency-discriminability model is inappropriate because the data are roughly linear—then the cubic coefficient will be positive and the lin-

Table 3

Cubic, quadratic, and linear coefficients of the least squares third-order polynomial fits to residuals for each model as a function of log obtained reinforcer ratio.

Pigeon	Generalized matching law				Contingency discriminability			
	Cubic	Quadratic	Linear	r^2	Cubic	Quadratic	Linear	r^2
26								
Large sample	0.005	-0.010	-0.21	.53	0.008	-0.011	-0.41	.74
Last five sessions	0.002	-0.003	-0.09	.19	0.006	-0.002	-0.31	.70
Davison-Jones	0.003	0.018	-0.14	.36	0.008	0.019	-0.41	.71
27								
Large sample	0.000	0.024	-0.04	.56	0.002	0.026	-0.18	.74
Last five sessions	0.006	0.029	-0.25	.60	0.003	0.024	-0.23	.52
Davison-Jones	0.003	0.026	-0.10	.42	0.006	0.027	-0.27	.57
299								
Large sample	0.003	0.010	-0.11	.78	0.006	0.010	-0.29	.97
Last five sessions	-0.001	0.013	0.06	.45	0.004	0.015	-0.20	.35
Davison-Jones	0.005	-0.001	-0.18	.84	0.007	-0.001	-0.31	.91
973								
Large sample	0.006	-0.008	-0.31	.93	0.008	-0.006	-0.52	.97
Last five sessions	0.006	0.013	-0.32	.89	0.007	0.013	-0.51	.94
Davison-Jones	0.000	-0.006	-0.02	.03	0.004	-0.006	-0.30	.23

ear coefficient will be negative. The lower right panel of Figure 4 shows the contingency-discriminability residuals for Pigeon 973's large samples. The polynomial was fitted just to the triangles (independent schedules with no COD). The curve conforms to the pattern shown at the upper right, indicating that the data deviate systematically from the model in just the way that would be expected if the data were approximately linear. Unlike the results of Davison and Jones (1995), the residuals representing the most extreme reinforcement ratios revealed none of the tendency toward less extreme choice required by the contingency-discriminability model. The additional points for dependent schedules and COD are in keeping with the rest, indicating again that these procedural variations had no systematic effect on choice.

Table 3 shows the results of polynomial fits like that shown in Figure 4 for all the pigeons and all the data sets. The contingency-discriminability residuals for all 4 birds in all three analyses showed patterns like that in the right graphs in Figure 4: positive cubic coefficients and negative linear coefficients. The quadratic coefficients were often different from zero, but varied unsystematically. Table 3 confirms for all birds and analyses the conclusion drawn for Pigeon 973's large sam-

ples: The choice ratios deviated from the contingency-discriminability model in the pattern that indicates that they approximated linearity throughout the range. The generalized-matching coefficients generally disconfirm any such pattern as that shown on the left in Figure 4. Except for the zero cubic coefficient for Pigeon 27's large samples, the pattern actually resembles that for the contingency-discriminability residuals. Although the choice ratios deviated systematically from the linearity of generalized matching, the pattern was opposite to that shown on the left in Figure 4. This deviation remains to be explained; we shall return to it below.

Changing over may be analyzed either by calculating frequency of switching, measured by rate or probability, or by calculating duration of visit, measured by time per visit or responses per visit. In the absence of visit-by-visit measures (e.g., Baum & Aparicio, 1999; Buckner et al., 1993), duration of visit was estimated by dividing number of pecks at an alternative by half the number of changeovers. Probability of switching was the reciprocal of this: half the number of changeovers divided by the number of pecks. This analysis was done only for the large samples.

Figure 5 shows pecks per visit (PPV) as a function of preference (base 2 log of the ratio

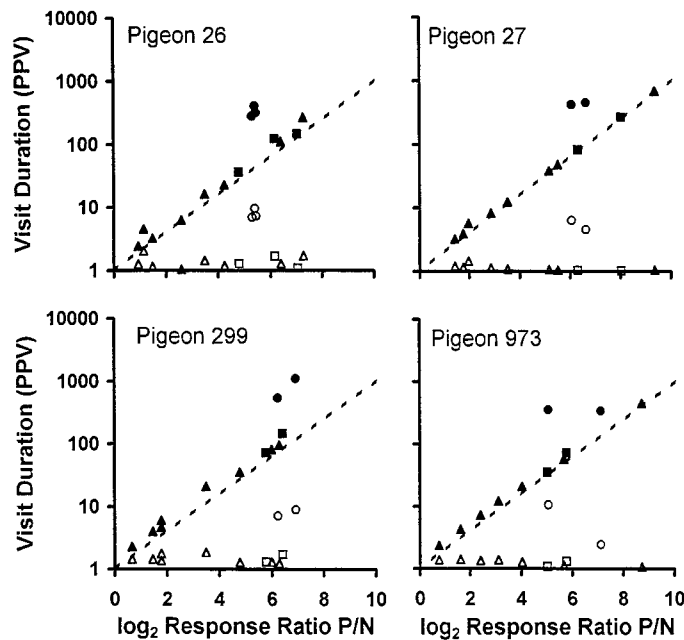


Fig. 5. Visit duration, measured as average pecks per visit (PPV), as a function of preference, measured as log (base 2) ratio of responses, preferred (rich) alternative (P) to nonpreferred alternative (N). Triangles represent conditions with independent schedules and no COD. Squares represent conditions with dependent scheduling and no COD. Circles represent conditions with dependent scheduling and a COD. Filled symbols represent the preferred alternative; open symbols represent the nonpreferred alternative. The broken line in each graph represents the locus of equality. Note logarithmic (base 10) y axis.

of pecks at the preferred alternative to pecks at the nonpreferred alternative). The absence of significant position bias allowed the results to be represented without regard to which side was preferred; when plotted separately, the results for the two sides were symmetrical. For the conditions with no COD, as soon as preferences exceeded indifference (i.e., a log value of zero)—even for preferences as small as 2:1—duration of visits to the nonpreferred side became brief, close to one peck, and showed no systematic variation with preference. Also for those conditions, log duration of visits on the preferred side increased linearly with preference, following the major diagonal. This indicates that pecks per visit on the preferred side almost equaled preference, an equality that would be perfect if the pigeons always made just one peck in a visit to the nonpreferred side. The points lie slightly above the major diagonal, however, suggesting a small systematic deviation from equality. Adding the 3-s COD increased PPV on both sides, producing longer visits to the

nonpreferred side but fewer of them, with the result that the COD had little effect on preference.

Figure 6 shows, for the conditions without a COD, log (base 2) preference as a function of log (base 2) reinforcement ratio, preferred side to nonpreferred side, regardless of position. For all 4 birds, the slope of the regression line was close to 1.0. In other words, when position (i.e., left-right) bias was ignored, the results approximated matching with bias. The negative values of log b indicate that there was a roughly constant proportional bias in favor of the nonpreferred (lean) alternative, regardless of whether it was on the left or the right.

The approximate matching shown in Figure 6 contrasts with the undermatching shown in Figure 1. The difference arises from the standard analysis's reliance on position as the defining characteristic of the alternatives, in contrast with the reliance in Figure 6 on rich versus lean or preferred versus nonpreferred, regardless of position. The approxi-

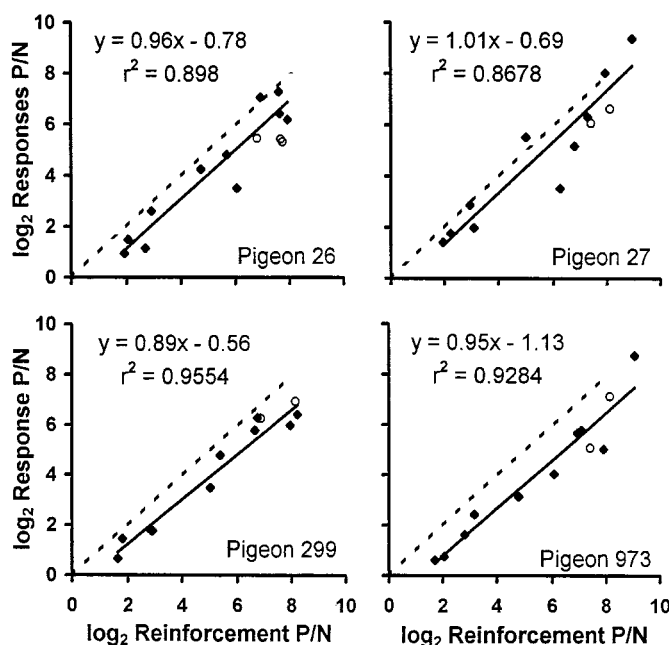


Fig. 6. Preference, measured as log (base 2) response ratio, preferred alternative to nonpreferred alternative, as a function of relative reinforcement, measured as log (base 2) reinforcement ratio, preferred alternative to nonpreferred alternative. The diamonds represent conditions without a COD. The circles represent conditions with a COD. The solid lines (equations shown) were fitted to the diamonds by the method of least squares. The broken lines represent perfect matching.

mate linearity of the relations in Figure 6 implies a systematic deviation from the standard matching relation defined in terms of position. If choice is governed by the difference between rich versus lean instead of left versus right, and there is a constant proportional bias toward the nonpreferred (lean) alterna-

tive, as shown in Figure 6, then that bias, when expressed in terms of position, will go in opposite directions, depending on whether the rich alternative is on the left or on the right.

Figure 7 shows, by way of illustration, this implication of Figure 6. The data of Pigeon 973 are replotted from Figure 1, except that two lines of slope 1.0 were fitted to the ratios: one for ratios less than 1.0 and one for ratios greater than 1.0. A horizontal line at indifference connects the two lines. The broken line shows the locus of strict matching. If the rich alternative was on the right (points to the lower left), the bias favored the left, and all points lay above the matching line, whereas if the rich alternative was on the left (points to the upper right), the bias favored the right, and all points fell below the matching line. As Figure 1 shows, a line fitted to all the points at once has a slope less than 1.0. Figure 7 shows, however, that choice deviated systematically from such a single line. As long as the relations in Figure 6 have slopes approximating 1.0, they imply the two-line pattern shown in Figure 7.

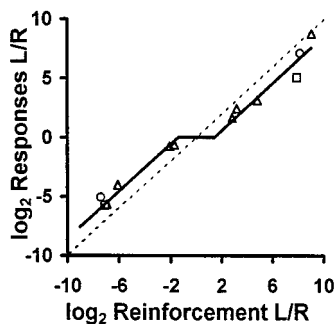


Fig. 7. The two-line relation implied by Figure 6 illustrated for Pigeon 973's choice relation (large samples; Figure 1). The two line segments, joined at indifference, have slopes equal to 1.0. The displacements (bias) from the broken line, the locus of strict matching, were derived by the method of least squares. Bias favors the lean alternative, whether on the left or the right.

Were a single line fitted to the points in Figure 7, as in Figure 1, the two-line pattern implies a specific pattern of deviation in the residuals. The residuals would be negative at the extreme left, then positive for points at the left nearer the middle, then negative again for points at the right near the middle, and then positive again for points at the extreme right. Inspection of the triangles in the graph for Pigeon 973 in Figure 1 verifies this pattern. It happens to be qualitatively the same pattern as shown in the upper right graph of Figure 4. It explains the similarity of coefficients for the two models shown in Table 3. If the data conform to the two-line pattern shown in Figure 7, then the generalized-matching residuals will approximately follow the pattern on the right in Figure 4, with the result that the cubic coefficient will be positive and the linear coefficient will be negative. Thus, the prevalence of this pattern in Table 3 further verifies the presence of the two-line pattern in all the birds' data and almost all of the data sets.

DISCUSSION

The results support a view of concurrent performance similar to that suggested by Houston and McNamara (1981): responding predominantly at the rich alternative occasionally interrupted by visits to the lean alternative that are brief and constant. Such a view contradicts any model of concurrent performance that assumes, explicitly or implicitly, that responding adjusts at both alternatives—for example, melioration and the kinetic model (Herrnstein & Vaughan, 1980; Myerson & Miezin, 1980). It may be reconciled with various optimality models, such as momentary maximizing, hill climbing, and global optimality (Baum, 1981; Hinson & Staddon, 1983; Houston & McNamara, 1981; Shimp, 1966). Even though the results go against the usual version of the generalized matching law, based on position or stimulus of the alternatives, they provide no support for the contingency-discriminability model. The contingency-discriminability model may account for more of the variance in the data obtained under procedures in which the stimuli associated with VI schedules are difficult to discriminate (e.g., Davison, 1996; Davison & Jones, 1995). However, under more typical

procedures it accounts for less variance in choice than does the generalized matching law (Figure 1), it deviates systematically from the choice relation (Figure 4 and Table 3), and it gains no support from the rich-lean analysis shown in Figure 6.

The lack of any systematic effect of dependent scheduling or adding a COD, shown in Figures 1, 2, and 3, rules out these procedural features as producing the difference in results between this experiment and that of Davison and Jones (1995). Another important difference between their methods and the present ones might have been the stability criterion used and the number of sessions completed per condition. Todorov, Castro, Hanna, Bittencourt de Sa, and Barreto (1983) reported effects of both number of conditions and number of sessions per condition. Although they found within-subject sensitivity to relative rates of reinforcement to decrease with increasing number of conditions, neither the present experiment nor that of Davison and Jones found any such effect of number of conditions. The finding by Todorov et al. that running more sessions per condition increased sensitivity, however, may apply to the present comparison. The Davison-Jones stability criterion caused conditions to change sooner in their experiment (after a minimum of 18 and a maximum of 37 sessions). Had their criterion been applied in this experiment, every condition would have ended far sooner than it did (after 18 to 34 sessions). Thus the large numbers of sessions completed per condition in the present study may explain why the sensitivity estimates were reduced for 3 of the 4 birds when the Davison-Jones stability criterion was applied (Table 2).

The small samples used by Davison and Jones (1995) also resulted in fewer than five reinforcers obtained on the lean alternative in the most extreme ratio conditions. In some conditions only one or two reinforcers were obtained on the lean schedule. Such small samples would increase the variability in estimates of relative responding and relative reinforcement. This inaccuracy is reflected in the variability of the choice relations shown in Table 2. For both analyses with small samples, the range of variation in sensitivity was greater than for the large samples. This is probably due mainly to the small numbers of

reinforcers in the small samples, because when only one or two reinforcers occur on the lean alternative, the reinforcement ratio may be highly volatile. When rate of reinforcement at the lean alternative is low, accuracy demands that the sample size for measuring it must be proportionately large.

Although the liberal stability criterion and the small samples may have reduced sensitivity and accuracy of estimates of choice and relative reinforcement, it is unlikely that they produced the systematic deviations from the generalized matching law that Davison and Jones (1995) observed. One cannot rule out the possibility that the volatile small samples deviated systematically by chance, but it seems more likely that the difference arose from their having used stimuli that were difficult to discriminate. The present experiment used two different locations, a difference known to produce good discriminations in pigeons, whereas Davison and Jones used two different brightness levels on the same key. If this difference is the cause, the contingency-discriminability model would apply only when the stimuli associated with the two alternatives were poorly discriminated and would have no bearing on typical procedures with highly discriminable stimuli.

Both the generalized matching law and the contingency-discriminability model may be reconciled with the present data by reexpressing them in terms of rich versus lean alternatives instead of position or stimuli. The matching law becomes

$$\log\left(\frac{B_P}{B_N}\right) = \log\left(\frac{r_P}{r_N}\right) + \log c, \quad (6)$$

where P and N refer to preferred (rich) and nonpreferred (lean) and c refers to bias favoring the rich or lean alternative. This approximates the lines fitted in Figure 6. It includes no sensitivity parameter, because the slopes were close to 1.0. The contingency-discriminability model (Equation 5) would reduce to Equation 6 if the subscripts relating to stimuli were replaced with P and N, and p were zero. It might then apply to procedures in which the alternatives' stimuli are imperfectly discriminated, if it could be shown that values of p greater than zero allowed correct fits to the response ratios:

$$\log\left(\frac{B_P}{B_N}\right) = \log\left(\frac{r_P - pr_P + pr_N}{r_N - pr_N + pr_P}\right) + \log c. \quad (7)$$

This is constructive because it allows us to account for two types of undermatching: that resulting from stimulus confusion and that associated with the generalized matching law. It suggests, however, that the latter type may be more apparent than real, because generalized-matching undermatching arises from fitting an inappropriate equation.

Although Figure 1 showed that fits of the response ratios to generalized matching were excellent when assessed by r^2 , goodness of fit cannot be assessed only by r^2 . Even if the variance accounted for is high, the remaining variance should be unsystematic. An equation may be shown to be incorrect by showing that the data deviate from it systematically. The residual patterns revealed in the coefficients in Table 3 demonstrate that the response ratios deviated systematically from the single line of generalized matching and instead conformed to a two-line pattern in which matching held throughout, but with bias in favor of the nonpreferred (lean) alternative. Figure 7 illustrated this two-line pattern for response ratios plotted in the usual way, according to position. Examination of Figure 6 also reveals that Equation 6 fitted the data with no systematic deviation. One would conclude that typical undermatching arises from characterizing concurrent performance incorrectly, because performance moves toward predominant responding at the rich alternative and occasional brief visits to the lean alternative (Figure 5).

Although such a view of concurrent performance allows Equation 6 to apply, it also suggests that an alternative analysis, focused on visits to the lean alternative, would be more appropriate. If we assume that visits to the nonpreferred alternative are brief and constant on average, which Figure 5 showed to be approximately true, then we may rewrite response ratios according to

$$\frac{B_P}{B_N} = \frac{B_P}{N \cdot D}, \quad (8)$$

where B_P and B_N are responses at the preferred and nonpreferred alternatives, N is the number of visits to the nonpreferred alternative, and D is the number of responses

made on a visit to the nonpreferred alternative. If D is close to 1.0, as in Figure 5, Equation 8 states that the response ratio will be close to the average duration (number of responses) of a visit to the preferred alternative. This result appears in Figure 5 in the close approximation of the triangles and squares to the major diagonal.

If we take the logarithm of Equation 8 and rearrange, we obtain

$$\log\left(\frac{B_P}{N}\right) = \log\frac{B_P}{B_N} + \log D, \quad (9)$$

the equation that would describe the points that parallel the major diagonal in Figure 5. This equation was used to estimate $\log D$ and D . For Pigeons 26, 27, 299, and 973, the estimates of $\log_2 D$ equaled 0.437, 0.152, 0.538, and 0.321. These correspond to estimates of D of 1.35, 1.11, 1.45, and 1.25—averages of a little more than one peck per visit to the nonpreferred lean alternative.

If we use Equation 8 to rewrite Equation 6, we obtain

$$\log\left(\frac{B_P}{N \cdot D}\right) = \log\left(\frac{r_P}{r_N}\right) + \log c. \quad (10)$$

Equation 10 approximates relations fitted to the points in Figure 6. The negative values of $\log c$ in Figure 6, which represent a value of c less than 1.0, indicate a bias in favor of the lean alternative. In contrast with the bias b of the generalized matching relation (Equation 4), which is assumed to be independent of reinforcement rates (r_1 and r_2), the bias c depends on reinforcement. It represents neither position bias nor stimulus bias, but rather favoritism toward the rich or lean alternative, regardless of position or stimulus.

Equation 10 implies that the bias shown in Figure 6 should be attributed to an excessive tendency to visit the lean alternative. The value of c indicates the proportion of N that should have been the number of visits to the lean alternative for strict matching to obtain. For example, the value of $\log_2 c$ for Pigeon 973, which was about -1.0 , would indicate that the obtained number of visits should be halved for strict matching; Pigeon 973 visited the lean side about twice as often as it would have according to perfect matching.

Thus, the apparent undermatching seen in Figure 1 and the good approximation to the

two-line pattern seen in Figure 7 and implied by the coefficients in Table 3 arose from an excessive tendency to visit the lean alternative. Such an explanation implies that penalties on switching, such as COD and travel, would increase apparent sensitivity (s) by decreasing the frequency of visiting the nonpreferred lean alternative, thereby increasing c . If c increased to 1.0, strict matching would result. If c increased beyond 1.0, as occurs with travel requirements, the generalized matching law would represent that bias—now in favor of the rich alternative—as overmatching (Baum, 1982; Boelens & Kop, 1983). In the present experiment, it appeared that the COD had two effects: It decreased the frequency of switching and increased the number of pecks made in a visit to the lean side (Figure 5). The consistency of the relative measures for the conditions including a COD with the rest (Figures 1 and 6) indicates that the two effects tended to cancel each other out.

If the choice relation is determined by the frequency of visits to the lean alternative, then analysis should focus on rate or probability of visiting the lean alternative as the key dependent variable. The present results point in that direction, as do some theoretical treatments of concurrent performance (e.g., Houston & McNamara, 1981) and some experimentation (e.g., Baum, 1982). If we rearrange Equation 10, we obtain

$$\log\left(\frac{N}{B_P}\right) = \log\left(\frac{r_N}{r_P}\right) - \log D - \log c, \quad (11)$$

which states that the probability of visiting the nonpreferred (lean) alternative depends directly on the ratio of reinforcement (lean to rich) with an intercept equal to $-(\log D + \log c)$ or bias equal to $1/Dc$. If B_P were measured as time spent responding, then N/B_P would represent rate of visiting the lean alternative instead of probability.

Figure 8 shows the application of Equation 11 to the large samples. The log (base 2) of the probability of visiting the lean alternative—the reciprocal of visit duration (Figure 5)—is shown as a function of the log (base 2) of the ratio of reinforcement, nonpreferred (lean) to preferred (rich). The fits are all good, with r^2 ranging from .86 to .97. All the slopes are substantially higher than for the

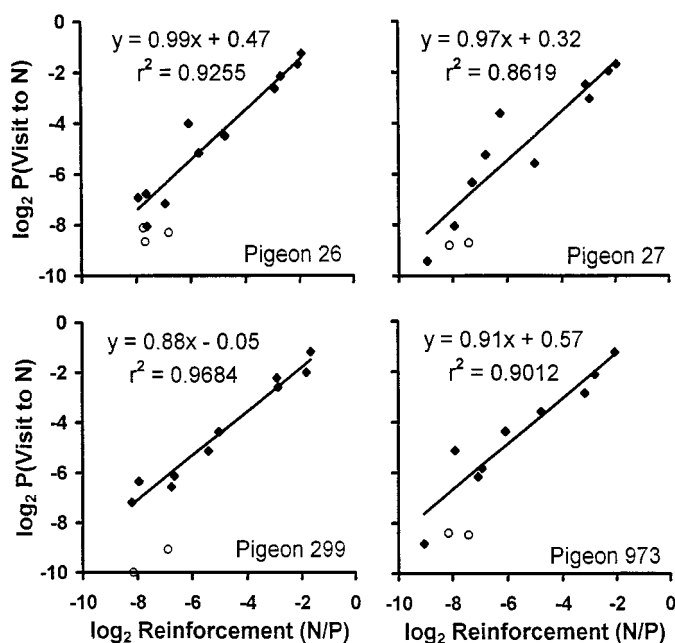


Fig. 8. Probability (log, base 2) of visiting the nonpreferred (lean) alternative as a function of relative reinforcement (log, base 2) at the nonpreferred alternative. Diamonds represent conditions with no COD. Circles represent conditions with a COD. The lines (equations shown) were fitted to the diamonds by the method of least squares.

standard analysis shown in Figure 1, and all are reasonable approximations to 1.0 (Baum, 1979). The slopes for the choice relations in Figure 6 are close to the slopes for probability of switching in Figure 8, as they should be. In sum, the probability of visiting the lean alternative was approximately directly proportional to (i.e., matched with bias) the relative reinforcement for the lean alternative. The points representing conditions with a COD tend to fall below the others, indicating that the COD decreased the probability of visiting the lean alternative. Although too few conditions with a COD were conducted, if the same matching relation held, the effect of the COD would be on the intercept of the relation, presumably by increasing visit duration D in Equation 11.

If we compare the intercepts in Figure 8 with those we derive from the estimates of D and c based on Figures 5 and 6, there is a rough correspondence. Pigeon 299, for which $\log_2 D$ and $\log_2 c$ were approximately equal but opposite in sign, shows an intercept close to zero. Pigeon 973, for which the difference between $\log_2 D$ and $\log_2 c$ was greatest, shows the largest intercept. The intercepts for

Pigeons 26 and 27, predicted from Equation 11 to be intermediate, are intermediate.

The standard analyses shown in Figure 1 suggested that choice relations that were uniform even out to reinforcement ratios over 200:1, but with undermatching. The analysis comparing the preferred (rich) alternative to the nonpreferred (lean) shown in Figures 5 and 6 restored an approximation to matching. At the same time, however, that analysis suggested a focus on visits to the rich and lean sides. Indeed, Figure 5 suggests that the systematic variance in choice arises solely from the systematic variance in visits at the rich alternative. Duration of visits to the rich alternative, however, is just the inverse of frequency of visiting the lean alternative. It thus becomes possible to characterize concurrent performance as responding on the rich alternative interrupted at some frequency, depending on the relative reinforcement there, by brief visits to the lean alternative (cf. Baum, 1982; Baum & Aparicio, 1999; Houston & McNamara, 1981; Houston *et al.*, 1987). When reinforcement approaches equality at the two alternatives, responding approaches indifference, and visits become

brief at both alternatives (i.e., rate of switching approaches a maximum). Figure 8 shows that such a characterization leads to an orderly and simple outcome, as simple as the original matching law (Herrnstein, 1961): Frequency of visiting the lean alternative is directly proportional to the relative reinforcement at the lean alternative. An earlier paper (Baum, 1982) argued that matching is a special case in the range of concurrent performances that can be generated by imposing penalties on switching. This view of concurrent performance may both explain variation in sensitivity and, at the same time, move us to an understanding more general than the matching law.

The findings in Figures 6 and 8 leave open a number of questions. The contingency-discriminability model (Equation 7) may apply to situations in which the concurrent alternatives are imperfectly discriminated. That remains to be tested. If the alternatives are highly discriminable, the present analysis suggests that concurrent performance consists of responding on the rich alternative occasionally interrupted by brief visits to the lean alternative. It remains to be seen how general these findings may be. For example, how does including a COD over a broad range of relative reinforcement affect the rich-lean relations shown in Figure 6 and the probabilities of visiting the lean alternative shown in Figure 8? In approaching such questions, it will be important to attend to the concerns addressed in the present methods, that is, to continue conditions until they are definitely stable and to gather large samples of stable performance. To draw quantitative conclusions, the data must be accurate and reliable.

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APPENDIX

Data for each subject summed over the large samples. Numbers in parentheses give the number of sessions in the sample.

Pigeon	Condition	Reinforcement ratio		Responses		Reinforcers		CO
		Programmed	Obtained	Left	Right	Left	Right	
26	1 (9)	4:1	3.8:1	13,145	6,900	569	151	10,900
	2 (11)	1:9	1:6.5	10,500	23,069	118	762	10,254
	3 (12)	64:1	66.1:1	30,246	2,680	2,049	31	3,756
	4 (9)	1:32	1:26.7	1,625	30,770	26	694	2,714
	5 (25)	128:1	199:1	61,488	723	1,990	10	1,125
	6 (56)	1:256	1:194	1,095	169,336	23	4,457	1,275
	7 (27)	4:1	4.2:1	46,328	16,628	1,743	417	28,599
	8 (12)	1:8	1:7.6	5,681	34,054	112	848	10,847
	9 (17)	64:1 d	51.3:1	47,751	1,711	1,334	26	2,660
	10 (57)	1:256 d	1:242	2,680	193,472	18	4,363	3,188
	11 (26)	128:1 d	121:1	79,139	597	2,063	17	1,099
	12 (38)	1:256 d, c	1:215	3,728	148,070	14	3,011	1,068
	13 (49)	128:1 d, c	111:1	153,459	3,505	3,885	35	968
	14 (64)	1:256 d, c	1:204	5,230	222,625	25	5,095	1,092
27	1 (7)	1:4	1:4.7	4,050	13,731	97	460	7,086
	2 (9)	9:1	8.6:1	20,736	5,266	645	75	7,344
	3 (23)	1:64	1:75.7	4,124	47,242	24	1,816	7,712
	4 (12)	32:1	32.1:1	35,660	780	931	29	1,488
	5 (18)	1:128	1:110	1,047	37,548	13	1,427	1,974
	6 (86)	256:1	490:1	218,538	334	6,866	14	640
	7 (9)	1:4	1:3.9	5,852	15,671	147	573	9,729
	8 (16)	8:1	7.8:1	28,727	3,942	1,135	145	6,991
	9 (31)	1:128 d	1:154	972	76,659	16	2,464	1,900
	10 (67)	256:1 d	243:1	190,044	732	5,338	22	1,438
	11 (38)	1:128 d, c	1:168	1,329	88,574	18	3,022	420
	12 (59)	256:1 d, c	277:1	159,014	1,615	4,703	17	706
299	1 (8)	1:4	1:3.1	7,815	12,338	155	485	10,888
	2 (13)	1:9	1:7.5	8,413	28,756	122	918	12,349
	3 (15)	64:1	41.9:1	57,818	2,097	1,172	28	3,278
	4 (15)	1:32	1:32.3	2,799	31,521	36	1,164	3,041
	5 (38)	128:1	107.6:1	141,766	1,827	3,012	28	2,975
	6 (93)	1:256	1:247	4,287	271,671	30	7,410	6,640
	7 (20)	4:1	3.5:1	51,931	18,911	1,246	354	26,121
	8 (19)	1:8	1:7.3	13,197	45,096	184	1,336	15,044
	9 (29)	128:1 d	100:1	112,017	2,048	2,297	23	3,150
	10 (56)	1:256 d	1:298	2,388	203,128	15	4,465	2,770
	11 (43)	128:1 d, c	118:1	172,863	2,289	3,411	29	640
	12 (50)	1:256 d, c	1:285	1,300	158,549	14	3,986	290
973	1 (9)	1:4	1:3.3	7,051	10,847	168	552	—
	2 (11)	9:1	9.1:1	14,217	2,646	793	87	3,965
	3 (30)	1:64	1:67.6	3,190	52,281	35	2,365	5,076
	4 (24)	32:1	27.7:1	29,241	3,345	1,853	67	4,860
	5 (23)	1:128	1:122	766	38,959	15	1,825	1,369
	6 (102)	256:1	532:1	138,469	325	7,985	15	614
	7 (16)	1:4	1:4.2	7,121	12,042	247	1,033	10,314
	8 (32)	8:1	7.1:1	28,572	9,272	2,244	316	13,320
	9 (32)	1:128 d	1:134	889	48,641	19	2,541	1,361
	10 (45)	256:1 d	239:1	61,808	1,907	3,585	15	3,536
	11 (34)	1:128 d, c	1:169	1,513	50,724	16	2,704	286
	12 (46)	256:1 d, c	282:1	77,265	557	3,667	13	458

Note. A d indicates dependent scheduling; a c indicates the use of a 3-s COD.